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BIANCHI TYPE-I (KASNER FORM) TRANSIT COSMOLOGICAL MODELS

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ABSTRACT

In this paper, a metric of the spatially homogeneous and anisotropic Bianchi type-I in Kasner form have been considered. The exact solutions of the Einstein's field equations with variable gravitational and cosmological constant have been obtained with the assumption of variation of scale factor, which yields a time dependent deceleration parameter. The gravitational constant $G(t)$ and cosmological constant $\Lambda(t)$ are also obtained. Physical behavior of the model has been discussed by obtaining some physical quantities.

Keywords: Bianchi type-I space-time in Kasner form, Gravitational constant and Cosmological constant.

INTRODUCTION

Einstein's field equations (with c = 1) contain two constants, gravitational constant (G) and cosmological constant (Λ). G plays the role of coupling constant between geometry and matter where as Λ stands for universal repulsion. G has many consequences in astrophysics. E Garcio-Berro *et al.* [1] and Belinchon [2] have studied the cosmological model in which G varies with time. Tripathi [3] has obtained Bianchi type-I universe with decaying vacuum energy and time varying gravitational constant.

The cosmological term, which is measure of the energy of empty space, provides a repulsive force opposing the gravitational pull between the galaxies. Recently, it is observed that smallness of cosmological constant impose problems in cosmology and elementary particle physics theory. In the absence of any interaction with matter or radiation, the cosmological constant remains a constant, but it is not true in general. In the presence of interactions with matter or radiation, a solution of Einstein equations and the assumed equation of covariant conservation of stress-energy with a time-varying Λ can be found. Recent observations by Perlmutter *et al.* [4],[5] and Riess *et al.* [6],[7] strongly favor a significant and a positive value of cosmological constant Λ. Kalita *et al.* [8],Chawla *et al.* [9] and Pradhan *et al.* [10] have studied the time dependent cosmological constant in different context. The observations of large-scale cosmic microwave background suggest that our physical universe is expanding isotropic and homogeneous models with a positive cosmological constant.

Ladke [11] has studied the Bianchi type-I (Kasner form) cosmological model in $f(R)$ theory of gravity. Recently, Pradhan *et al.* [12] have studied the Bianchi type-I transit cosmological models with time dependent gravitational and cosmological constants. Motivated by the above research, in this paper, considering the space-time metric of the spatially homogeneous and anisotropic Bianchi type-I in Kasner form, the exact solutions of the Einstein's field equations with variable gravitational and cosmological constant have been obtained with the consideration of law variation of scale factor which yield a time dependent deceleration parameter. Gravitational constant (G) and cosmological constant (Λ) are also obtained. The physical behavior of the model has been discussed.

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METRIC AND FIELD EQUATIONS

We consider the anisotropic and spatially homogeneous Bianchi type-I in Kasner form space-time metric as

$$ds^2 = -dt^2 + t^{2q_1}dx^2 + t^{2q_2}dy^2 + t^{2q_3}dz^2 , \quad (1)$$

where q_1, q_2 and q_3 are constant.

Einstein field equation with time dependent G and Λ are given by

$$R_{ij} - \frac{1}{2} g_{ij} R = -8\pi G(t) T_{ij} + \Lambda(t) g_{ij} , \qquad (2)$$

where R_{ij} is the Ricci tensor; R is the Ricci scalar, G is the gravitational constant and Λ is the cosmological constant.

For a perfect fluid, the stress energy-momentum tensor T_{ij} is given by

$$T_{ij} = (\rho + p) v_i v_j - p g_{ij} , \qquad (3)$$

where $\rho =$ matter density, $p =$ thermodynamic pressure, $v^i =$ fluid four-velocity vector of the fluid which satisfies

$$v^i v_j = 1 , \qquad (4)$$

Choosing comoving system of coordinates, equation (2) for the metric (1) and (3) becomes

$$\frac{\left(t^{q_1}\right)^{\cdot\cdot}}{t^{q_1}} + \frac{\left(t^{q_2}\right)^{\cdot\cdot}}{t^{q_2}} + \frac{\left(t^{q_1}\right)^{\cdot}}{t^{q_1}} \frac{\left(t^{q_2}\right)^{\cdot}}{t^{q_2}} = -8\pi G(t) p + \Lambda(t) , \qquad (5)$$

$$\frac{\left(t^{q_2}\right)^{\cdot\cdot}}{t^{q_2}} + \frac{\left(t^{q_3}\right)^{\cdot\cdot}}{t^{q_3}} + \frac{\left(t^{q_2}\right)^{\cdot}}{t^{q_2}} \frac{\left(t^{q_3}\right)^{\cdot}}{t^{q_3}} = 8\pi G(t) p + \Lambda(t) , \qquad (6)$$

$$\frac{\left(t^{q_1}\right)^{\cdot\cdot}}{t^{q_1}} + \frac{\left(t^{q_3}\right)^{\cdot\cdot}}{t^{q_3}} + \frac{\left(t^{q_1}\right)^{\cdot}}{t^{q_1}} \frac{\left(t^{q_3}\right)^{\cdot}}{t^{q_3}} = -8\pi G(t) p + \Lambda(t) , \qquad (7)$$

$$\frac{\left(t^{q_1}\right)^{\cdot}}{t^{q_1}} \frac{\left(t^{q_2}\right)^{\cdot}}{t^{q_2}} + \frac{\left(t^{q_2}\right)^{\cdot}}{t^{q_2}} \frac{\left(t^{q_3}\right)^{\cdot}}{t^{q_3}} + \frac{\left(t^{q_1}\right)^{\cdot}}{t^{q_1}} \frac{\left(t^{q_3}\right)^{\cdot}}{t^{q_3}} = -8\pi G(t) p + \Lambda(t) . \qquad (8)$$

The covariant divergence of equation (2) gives

$$\dot{\rho} + 3(\rho + p) H + \rho \frac{\dot{G}}{G} + \frac{\dot{\Lambda}}{8\pi G} = 0 . \qquad (9)$$

The spatial volume V for Bianchi type-I universe in Kasner form is

$$V = t^S . \qquad (10)$$

We define a meam scale factor a as

$$a = V^{1/3} = t^{S/3} . \qquad (11)$$

The mean Hubble parameter H is defined as

$$H = \frac{1}{3}\left(H_1 + H_2 + H_3\right) , \qquad (12)$$

where $H_1 = \frac{q_1}{t}$, $H_2 = \frac{q_2}{t}$, $H_3 = \frac{q_3}{t}$ are the directional Hubble parameters in the directions of x, y, z respectively and dot (·) denotes derivative with respect to cosmic time t.

From (11) and (12), we get

$$H = \frac{\dot{a}}{a} = \frac{1}{3} \frac{S}{t} , \qquad (13)$$

where $S = q_1 + q_2 + q_3$.

The dynamic scalars that are of cosmological importance are

The expansion scalar is given by

$$\theta = 3H = \frac{S}{t} . \qquad (14)$$

The mean anisotropy parameter is given by

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 . \tag{15}$$

The shear scalar σ is defined as

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^2 - 3H^2\right)$$

$$\sigma^2 = \frac{1}{2}\left[\left(\frac{(t^{q_1})^{\cdot}}{t^{q_1}}\right)^2 + \left(\frac{(t^{q_2})^{\cdot}}{t^{q_2}}\right)^2 + \left(\frac{(t^{q_3})^{\cdot}}{t^{q_3}}\right)^2\right] - \frac{\theta^2}{6} . \tag{16}$$

We define a deceleration parameter as

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -\left(\frac{\dot{H} + H^2}{H^2}\right). \tag{17}$$

SOLUTIONS OF THE FIELD EQUATIONS

The field equations (5)-(8) are a system of four equations with seven unknowns $q_1, q_2, q_3, G, p, \rho$ and Λ. Hence, to obtain explicit solution of the system three more condition are required.
First, we assume the power-law form of the gravitational constant *G*.

$$G \propto a^m \text{, i.e. } G = h a^m , \tag{18}$$

where h is the constant of proportionality and is a positive constant and *m* is constant.
Secondly, we assume equation of state for perfect fluid

$$p = \gamma\rho , \tag{19}$$

where $\gamma(0 \le \gamma \le 1)$ is a constant.
Now, subtracting equations (6) from (7), (7) from (5) and (5) from (6), we get

$$\frac{\left(t^{q_1}\right)^{\cdot\cdot}}{t^{q_1}} - \frac{\left(t^{q_2}\right)^{\cdot\cdot}}{t^{q_2}} + \frac{\left(t^{q_3}\right)^{\cdot}}{t^{q_3}}\left[\frac{\left(t^{q_1}\right)^{\cdot}}{t^{q_1}} - \frac{\left(t^{q_2}\right)^{\cdot}}{t^{q_2}}\right] = 0 , \tag{20}$$

$$\frac{\left(t^{q_2}\right)^{\cdot\cdot}}{t^{q_2}} - \frac{\left(t^{q_3}\right)^{\cdot\cdot}}{t^{q_3}} + \frac{\left(t^{q_1}\right)^{\cdot}}{t^{q_1}}\left[\frac{\left(t^{q_2}\right)^{\cdot}}{t^{q_2}} - \frac{\left(t^{q_3}\right)^{\cdot}}{t^{q_3}}\right] = 0 , \tag{21}$$

$$\frac{\left(t^{q_3}\right)^{\cdot\cdot}}{t^{q_3}} - \frac{\left(t^{q_1}\right)^{\cdot\cdot}}{t^{q_1}} + \frac{\left(t^{q_2}\right)^{\cdot}}{t^{q_2}}\left[\frac{\left(t^{q_3}\right)^{\cdot}}{t^{q_3}} - \frac{\left(t^{q_1}\right)^{\cdot}}{t^{q_1}}\right] = 0 . \tag{22}$$

These equations imply that

$$\frac{t^{q_1}}{t^{q_2}} = K_1 \exp\left(n_1 \int \frac{dt}{V}\right), \tag{23}$$

$$\frac{t^{q_2}}{t^{q_3}} = K_2 \exp\left(n_2 \int \frac{dt}{V}\right), \tag{24}$$

$$\frac{t^{q_3}}{t^{q_1}} = K_3 \exp\left(n_3 \int \frac{dt}{V}\right), \tag{25}$$

where K_1 , K_2 , K_3 and n_1 , n_2 , n_3 are constants of integration.
After solving equations (23)-(25), we get

$$t^{q_1} = t^{S/3} m_1 \exp\left(r_1 \int \frac{dt}{V} \right) , \quad (26)$$

$$t^{q_2} = t^{S/3} m_2 \exp\left(r_2 \int \frac{dt}{V} \right) , \quad (27)$$

$$t^{q_3} = t^{S/3} m_3 \exp\left(r_3 \int \frac{dt}{V} \right) , \quad (28)$$

where $m_1 = (K_1 K_2)^{\frac{1}{3}}$, $m_2 = (K_1^{-1} K_3)^{\frac{1}{3}}$, $m_3 = (K_2^{-1} K_3^{-1})^{\frac{1}{3}}$ and (29)

$$r_1 = \frac{n_1 + n_2}{3} , \; r_2 = \frac{n_3 - n_1}{3} \; , r_3 = -\frac{(n_2 + n_3)}{3} . \quad (30)$$

Here r_1 , r_2 , r_3 and m_1 , m_2 , m_3 satisfy the relation

$$r_1 + r_2 + r_3 = 0 \quad , \quad m_1 m_2 m_3 = 1 . \quad (31)$$

Now we can obtain the metric function as a function of t if mean scale factor is known

Here we consider $a = \sqrt{t^n e^t}$ (32)

If we put $n = 0$ in above equation then $a = \sqrt{e^t}$ i.e. an exponential law of variation of scale factor. This choice of scale factor yield a time-dependent deceleration parameter. The time dependent deceleration parameter the universe is accelerated expansion at present as observed in recent observation of type I_a supernova and decelerated expansion in the past.

Using equation (32) in equation (26), (27),(28), we get

$$t^{q_1} = m_1 (t^n e^t)^{1/2} \exp\left[r_1 \int (t^n e^t)^{-3/2} dt \right] , \quad (33)$$

$$t^{q_2} = m_2 (t^n e^t)^{1/2} \exp\left[r_2 \int (t^n e^t)^{-3/2} dt \right] , \quad (34)$$

$$t^{q_1} = m_1 (t^n e^t)^{1/2} \exp\left[r_3 \int (t^n e^t)^{-3/2} dt \right] , \quad (35)$$

Solving these equations, we obtain

$$t^{q_1} = m_1 (t^n e^t)^{1/2} \exp[r_1 I(t)] , \quad (36)$$

$$t^{q_2} = m_2 (t^n e^t)^{1/2} \exp[r_2 I(t)] , \quad (37)$$

$$t^{q_3} = m_3 (t^n e^t)^{1/2} \exp[r_3 I(t)] , \quad (38)$$

where $I(t) = \int (t^n e^t)^{-3/2} dt = \sum_{i=1}^{\infty} \frac{(-3)^{i-1} t^{i - \frac{3n}{2}}}{2^{i-2}(2i - 3n)(i-1)}$. (39)

Hence space-time metric reduced to

$$ds^2 = -dt^2 + m_1 (t^n e^t) \left[m_1^2 \exp\left(2 r_1 I(t) dx^2\right) + m_2^2 \exp\left(2 r_2 I(t) dy^2\right) + m_3^2 \exp\left(2 r_3 I(t) dz^2\right) \right] .$$

(40)

Using equations (36)-(38), the directional Hubble parameters in the directions of x , y and z -axes are found to be

$$H_1 = \frac{1}{2}\left(1 + \frac{n}{t}\right) + r_1 (t^n e^t)^{-3/2} , \quad (41)$$

$$H_2 = \frac{1}{2}\left(1 + \frac{n}{t}\right) + r_2 (t^n e^t)^{-3/2} , \quad (42)$$

$$H_3 = \frac{1}{2}\left(1+\frac{n}{t}\right) + r_3\left(t^n e^t\right)^{-3/2}. \tag{43}$$

The Mean Hubble parameter is given by

$$H = \frac{1}{2}\left(1+\frac{n}{t}\right). \tag{44}$$

Spatial volume V is given by

$$V = \left(t^n e^t\right)^{3/2}. \tag{45}$$

The expansion scalar $\theta = 3H$ is given by

$$\theta = \frac{3}{2}\left(1+\frac{n}{t}\right). \tag{46}$$

The mean anisotropy parameter Δ is given by

$$\Delta = \frac{4}{3}\left(r_1^2 + r_2^2 + r_3^2\right)\left(1+\frac{n}{t}\right)^{-2}\left(t^n e^t\right)^{-3}. \tag{47}$$

The shear scalar σ is given by

$$\sigma^2 = \frac{1}{2}\left(r_1^2 + r_2^2 + r_3^2\right)\left(t^n e^t\right)^{-3}. \tag{48}$$

The deceleration parameter q is given by

$$q = \frac{2n}{(n+1)^2} - 1. \tag{49}$$

Using equation (18) and (32), we obtain

$$G = h\left(t^n e^t\right)^{m/2}. \tag{50}$$

It is observed that G is an increasing function of time i.e. $G \to 0$ as $t \to 0$ and $G \to \infty$ as $t \to \infty$

Energy density, pressure and cosmological constant for universe given by

$$\rho = \frac{1}{8\pi h(1+\gamma)}\left[nt^{-(mn+4)/2}e^{-(mt)/2} - \beta_1\left(t^n e^t\right)^{-(m+6)/2}\right], \tag{51}$$

$$p = \frac{\gamma}{8\pi h(1+\gamma)}\left[nt^{-(mn+4)/2}e^{-(mt)/2} - \beta_1\left(t^n e^t\right)^{-(m+6)/2}\right], \tag{52}$$

$$\Lambda = \frac{3}{4}\left(1+\frac{n}{t}\right)^2 + \frac{1}{(1+\gamma)}\left[\beta_2\left(t^n e^t\right)^{-3} - nt^{-2}\right], \tag{53}$$

where $\beta_2 = m_1^2 + m_2^2 + m_1 m_2 + \gamma\left(m_1 m_2 + m_2 m_3 + m_3 m_1\right)$.

We find that above solutions satisfy equation (9) identically and hence represent exact solution of Einstein's field equation (5)-(8).

DISCUSSION AND CONCLUSION

i) From the equation (45), it is observed that as $t = 0$, the spatial volume vanishes. It expands exponentially as t increase and becomes infinitely large as $t \to \infty$.

ii) From equation (46), it is observed that, the expansion scalar θ starts with infinite value at $t = 0$ and then becomes constant after some finite time.

iii) From equation (47), it is observed that when time $t \to \infty$, we get $\Delta \to 0$ Thus, this model has transition from initial anisotropy to isotropy at present epoch.

iv) From the equation (49), we observe that $q > 0$ for $t < \sqrt{2n} - n$ and $q < 0$ for $t > \sqrt{2n} - n$. It is observed that for $0 < n < 2$, this model evolves from deceleration to acceleration phase.

v) From equation (50), we observe that G is an increasing function of time i.e. $G \to 0$ as $t \to 0$ whereas for $t \to \infty$, $G \to \infty$.

vi) From equation (53), we observe that Λ is a decreasing function of time and it approaches to a small positive value at late time.

vii) It is interesting to note that all the results obtained by Pradhan *et al*. ((2014) can be reproduced from these results by giving appropriate values to the functions concerned.

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